Name: _____

IC No.: _____

Seat No: _____

BCA ACADEMY

SCHOOL OF BUILDING & DEVELOPMENT

SINGAPORE

MATHEMATICS SCREENING TEST (SET K)

1.5 HOURS

Instructions to candidates

1. Do **not** turn over this page until you are told to do so.
2. Check that you have the correct exam paper, number of pages and questions.
3. This paper consists of **TEN (10)** questions (100 marks). Answer ALL questions
4. Write your **Name, IC NO. and Seat No.** on this cover page.
5. All answers are to be written in THIS booklet.
6. Do **NOT** tear out any page. This booklet is the property of BCA Academy and **must not be removed** from the test centre.
7. All mobile phones and electronic equipment are to be switched off.
8. Candidates are to bring their own non-programmable scientific calculator.
   - Unless otherwise stated, leave your answers in 3 significant figures.
   - Unless the questions require the answers in term of π, the calculator value for $\pi = 3.142$ should be used.
   - If working is needed for any question, it must be shown with the answer. Omission of essential working will result in loss of marks.

For Official Use:	Test Centre:	Test Date:	Marks(/100):
	Marker:	Checker:	

1. Simplify the following expressions: (10 marks)

(a) $2 [3(2x - 3) + 4(x + 1)] / [3(2x -1)]$

(b) $0.5 [3(5 - 2) - 4 (1 + 2)] / [2(1 + 3)]$

2. Simplify the following equations: (10 marks)

(a) $$\frac{(25 - c^2)(1 + c)}{3(5 + c)(1 - c^2)}$$

(b) $ab(a^2b - ab^2) + (ab)^2(a + b)$

3. As shown in the diagram, an ice cream cone consists of a rolled wafer in the shape of a cone of length $3x$ and height h. On top of the cone is a scoop of ice cream in the shape of a hemisphere of radius x. All dimensions are in cm.

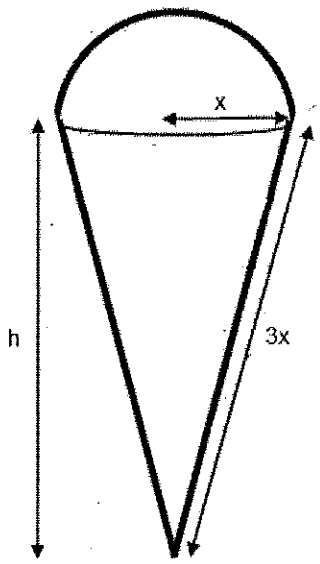
Given that the total external surface area of the ice cream cone is 300 cm^2, find:

(a) The value of x. (5 marks)

(b) The value of h (5 marks)

Formulae: For a cone of radius r and length l, curved surface area of a cone = $2\pi rl$

For a hemisphere of radius r, surface area of a hemisphere = $2\pi r^2$

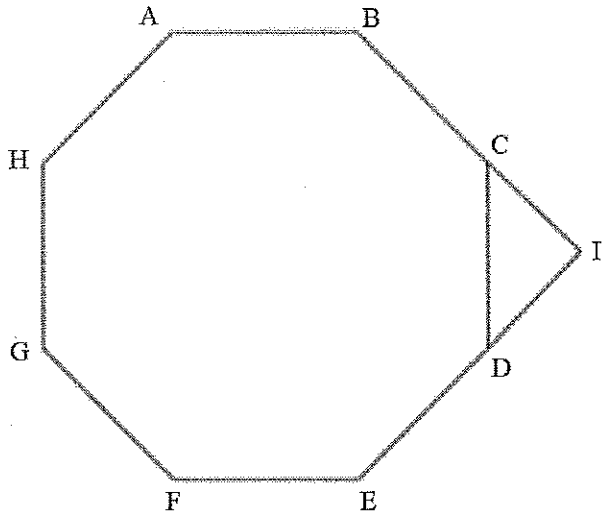


4. ABCDEFGH is a regular 8-sided polygon. Sides BC and DE have been extended as shown to form the isosceles triangle CID. Find:

(a) $\angle BCD$ (5 mark)

(b) $\angle DCI$ (2 marks)

(c) $\angle CID$ (3 marks)



5. In the diagram, WX and XY are two equal tangents from the external point X. Angle WXY is equal to 40°. Find:

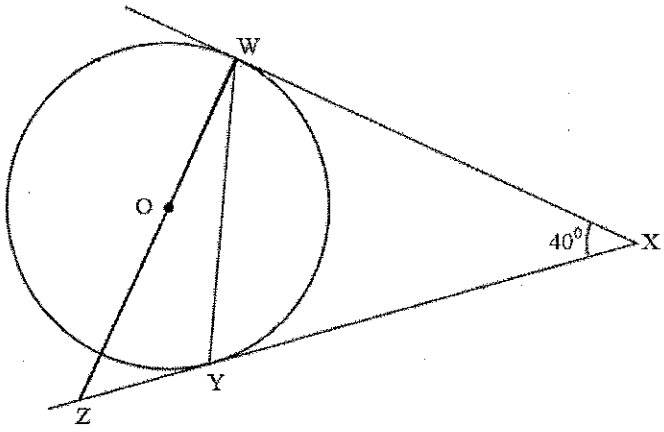
(a) ∠OWX (1 mark)

(b) ∠XWY (2 marks)

(c) ∠OWY (2 marks)

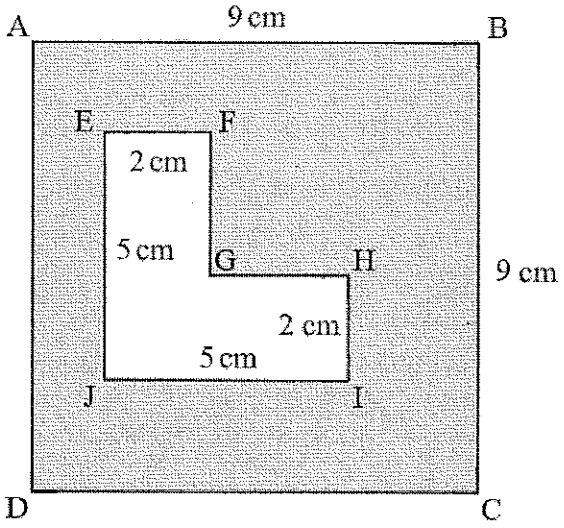
(d) ∠WYZ (3 marks)

(e) ∠WZY (2 marks)



6. Jim is two years older than Bob, who is twice the age of Alan. The sum of all their ages is 127. Find the ages of Jim, Bob, and Alan. (10 marks)

7. ABCD is a square piece of cardboard of dimensions 9 cm x 9 cm. An L-shaped section EFGHIJ has been cut out of the cardboard.



Calculate

(a) The area of the shaded region. (5 marks)

(b) The perimeter of the shaded region. (5 marks)

8. A bus leaves Town A and travels towards Town B at a constant speed. Five hours later, a car leaves Town A at a constant speed of 80 km/h. The car catches up with the bus three (3) hours later. The car reaches Town B two (2) hours after it caught up with the bus.

(a) Find the speed of the bus. (3 marks)

(b) Calculate the distance between Town A and Town B. (5 marks)

(c) Calculate the total travel time (to the nearest hour) for the bus to travel from Town A to Town B. (2 marks)

9. A cookie jar contains six chocolate cookies, five vanilla cookies, four oatmeal cookies, and three raisin cookies. The jar is designed such that no one can look inside the jar and the cookies must be randomly drawn from the jar. Expressing all your answers in fractions,

(a) What is the probability of drawing one raisin cookie if only one cookie can be drawn? (2 marks)

(b) What is the probability of drawing three vanilla cookies if exactly three cookies can be drawn one after the other? (3 marks)

(c) What is the probability of drawing (in order) one chocolate cookie, one vanilla cookie, one oatmeal cookie, and one raisin cookie? (5 marks)

10. (a) Find the value of y if:

$$4^{y} = 3(2^{1}) + 2^{y}$$ (5 marks)

(b) It is given that:

$$\log_a (2) = 0.3456$$
$$\log_a (3) = 0.4398$$
$$\log_a (5) = 0.5567$$
$$\log_a (7) = 0.7865$$

Using this information, find the value of a if

$$3^{a} + \log_a (10) = \log_a (21)$$ (5 marks)

END OF PAPER

ANSWER KEY

1.

(a). $$\frac{2\left[3(2x-3)+4(x+1)\right]}{3(2x-1)}$$

$$= \frac{2\left[6x-9+4x+4\right]}{3(2x-1)}$$

$$= \frac{2(10x-5)}{3(2x-1)}$$

$$= \frac{2(5)(2x-1)}{3(2x-1)}$$

$$= \frac{10}{3}$$

(b) $$\frac{0.5\left[3(5-2)-4(1+2)\right]}{2\left[1+3\right]}$$

$$= \frac{0.5\left[3(3)-4(3)\right]}{2(4)}$$

$$= \frac{9-12}{2(2)(4)}$$

$$= \frac{-3}{16}$$

2.

2(a) $$\frac{(25-c^2)(1+c)}{3(5+c)(1-c^2)} = \frac{(5+c)(5-c)(1+c)}{3(5+c)(1+c)(1-c)}$$

$$= \frac{5-c}{3(1-c)}$$

(b) $ab(a^2b - ab^2) + (ab)^2(a+b)$

$= a^3b^2 - a^2b^3 + a^2b^2(a+b)$

$= a^3b^2 - a^2b^3 + a^3b^2 + a^2b^3$

$= 2a^3b^2$

3.

3 (a).

$$2\pi rl + 2\pi r^2 = 300$$

$$\pi rl + \pi r^2 = 150$$

$$\pi r(l + r) = 150$$

$$\pi(x)(3x + x) = 150$$

$$4\pi x^2 = 150$$

$$x = \sqrt{\frac{150}{4\pi}}$$

$$= 3.45 \text{ cm}.$$

(b) By Pythagoras' Theorem,

$$(3x)^2 = x^2 + h^2$$

$$9x^2 = x^2 + h^2$$

$$h^2 = 8x^2$$

$$h = x\sqrt{8}$$

$$= (3.45)\sqrt{8}$$

$$h = 9.77 \text{ cm}.$$

4.

4 (a). Interior angle $\angle BCD = \frac{180(n-2)}{n}$

$= \frac{180(8-2)}{8}$

$= 135^\circ$ (int $\angle$s of polygon)

(b) $\angle DCI = 180^\circ - 135^\circ = 45^\circ$ (supplementary $\angle$s)

(c). $\angle CID = 180^\circ - 45^\circ - 45^\circ$

$= 90^\circ$ (isosceles $\triangle$)

5.

(a) $\angle OWX = 90°$ (tangent to circle)

(b) $\angle XWY = \frac{180° - 40°}{2}$

$= 70°$ (isosceles $\triangle$)

(c) $\angle OWY = \angle OWX - \angle XWY$

$= 90° - 70°$

$= 20°$

(d) $\angle WYZ = 180° - \angle WYX$

$= 180° - \angle XWY$

$= 180° - 70°$

$= 110°$ (supplementary $\angle$s)

(e) $\angle WZY = 180° - \angle OWY - \angle WYZ$

$= 180° - 20° - 110°$

$= 50°$ ($\angle$s of a triangle)

6.

6. Let Alan's age be x.

Bob's age be $2x$.

Jim's age be $2x+2$.

Then, $x + 2x + 2x + 2 = 127$

$5x + 2 = 127$

$5x = 125$

$x = 25$

$\therefore$ Alan's age is 25.

Bob's age is 50.

Jim's age is 52.

7.

7 (a) Area of square ABCD = 9 × 9

= 81 cm²

Area of EFGHIJ = (2 × 3) + (2 × 5)

= 16 cm²

Shaded area = 81 − 16

= 65 cm²

(b) Perimeter of shaded region

= 9 + 9 + 9 + 9 + 2 + 3 + 3 + 2 + 5 + 5

= 56 cm

8.

8 (a) Travel time of bus at catch-up point,

= 5 + 3

= 8 hrs.

Distance travelled by bus / car at catch-up point

= 80 × 3

= 240 km

Speed of bus

$= \frac{240}{8}$

= 30 km/h

(b) Distance between Town A and Town B

= Distance travelled by car

= Speed × total time

= 80 × (5 + 3 + 2)

= 800 km

(c) Total time taken by bus

$= \frac{\text{Distance}}{\text{Time}}$

$= \frac{800}{30}$

= 26.7 hrs

= 27 hrs (nearest hr)

9.

9 (a) $P(\text{one raisin cookie}) = \frac{3}{(6+5+4+3)} = \frac{1}{6}$

(b) $P(\text{three vanilla cookies})$

$= \left(\frac{5}{(6+5+4+3)}\right) \times \left(\frac{4}{(6+4+4+3)}\right) \times \left(\frac{3}{(6+3+4+3)}\right)$

$= \frac{5}{18} \times \frac{4}{17} \times \frac{3}{16}$

$= \frac{5}{408}$

(c) $P(\text{one chocolate, one vanilla, one oatmeal, one raisin})$

$= \left(\frac{6}{6+5+4+3}\right) \times \left(\frac{5}{5+5+4+3}\right) \times \left(\frac{4}{5+4+4+3}\right) \times \left(\frac{3}{5+4+3+3}\right)$

$= \left(\frac{6}{18}\right) \times \left(\frac{5}{17}\right) \times \left(\frac{4}{16}\right) \times \left(\frac{3}{15}\right)$

$= \frac{1}{204}$

10.

10 (a) $4^y = 3(2^1) + 2^y$

$(2^y)^2 = 6 + 2^y$

x	-3	$-3x$
x	$+2$	$+2x$
x^2	-6	$-x$

Let $x = 2^y \Rightarrow x^2 = 6 + x$

$x^2 - x - 6 = 0$

$(x+2)(x-3) = 0$

$x = -2$ or $x = 3$

$\Rightarrow 2^y = -2$ (rejected) or $2^y = 3$

$y \ln 2 = \ln 3$

$y = \frac{\ln 3}{\ln 2}$

$= 1.58.$

b). $3^a + \log_a(10) = \log_a(21)$

$\Rightarrow 3^a + \log_a(5 \times 2) = \log_a(3 \times 7)$

$3^a + \log_a(5) + \log_a(2) = \log_a(3) + \log_a(7)$

$3^a + 0.5567 + 0.3456 = 0.4398 + 0.7865$

$3^a = 0.324$

$a \ln 3 = \ln(0.324)$

$a = \frac{\ln(0.324)}{\ln 3}$

$= -1.026$